Magnetoresistance oscillations in two-subband electron systems: Influence of electron-phonon interaction

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A theory of the quantum oscillations of resistivity due to interaction of two-dimensional electrons with impurities and acoustic phonons in the presence of perpendicular magnetic field is developed for multisubband systems. A comparison of the theory with recent experimental data for a quantum well with two occupied subbands demonstrates a good agreement and confirms that peculiar features of the observed magnetoresistance are caused by interference of magnetointersubband and phonon-induced oscillations. It is shown that the intersubband phonon-assisted transitions are responsible for this interference phenomenon.

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I. INTRODUCTION

Quantum oscillations of resistivity in high-mobility two-dimensional (2D) electron layers in the region of relatively weak perpendicular magnetic fields ($B \leq 1$ T) attract a considerable attention during the past decade. Apart from the well-known Shubnikov–de Haas oscillations (SdHO) which are caused by oscillations of the density of states at the Fermi level and strongly damped with increasing temperature T, there exists a class of oscillations which survive increase in T. These oscillations originate from scattering-assisted coupling of electron states in different Landau levels. If elastic scattering (by impurities, interface roughness, etc.) dominates, the coupling can be achieved by application of a strong dc voltage[1] or in the presence of microwave radiation.[2] Inelastic scattering by acoustic phonons enables transitions of electrons between Landau levels even in the linear transport regime. This scattering becomes important along with the elastic one for samples of high purity and when the temperature is increased. The probability of electron-phonon scattering is maximal when the phonon momentum Q is close to the Fermi circle diameter, $2p_F$. This sensitivity of the scattering to acoustic phonon momentum leads to a special kind of magnetophonon oscillations known as phonon-induced resistance oscillations (PIRO).[3–10] The periodicity of PIRO is governed by commensurability of the characteristic phonon frequency (determined by the transferred momentum $2p_F$) with cyclotron frequency ω_c. In GaAs quantum wells of very high mobility ($\sim 10^7$ cm^2/V s), these oscillations are visible starting from $T \simeq 2$ K.[8] Initially, PIRO was explained by involving the interface phonon model. Later, it was found that the model of bulk (three-dimensional) phonons gives similar results for magnetoresistance. A theory of PIRO based on consideration of 2D electrons interacting with anisotropic bulk phonon modes has been presented recently.[10]

Electron systems with two (or more) populated 2D subbands, which are realized in double quantum wells and wide single quantum wells with high electron density, have peculiar magnetoresistance properties because of the possibility of intersubband transitions of electrons. In the presence of elastic scattering, the intersubband transitions couple different Landau levels under the condition that subband splitting energy Δ is a multiple of the cyclotron energy. The resistivity of two-subband systems exhibits magnetointersubband oscillations[11] (MISO) owing to periodic modulation of the probability of intersubband scattering by the magnetic field. The MISO have been studied both experimentally[12–17] and theoretically[11,18–20] for more than two decades. Recent observations of MISO with large amplitudes in high-mobility double quantum wells[16,17] have stimulated attention to this phenomenon. A special interest is devoted to the interference of MISO with microwave-induced oscillations of magnetoresistance,[17,21] which is observed also in three-subband systems realized in triple quantum wells.[22] This is a representative of quantum interference phenomena caused by combined action of different mechanisms of electron transitions between Landau levels; the other examples can be found in Refs. 6, 23, and 24.

Recent experimental results[25] indicate that the interference phenomena in magnetoresistance of 2D systems with two populated subbands can be observed in the linear transport regime, without either a strong dc voltage or microwave illumination. A peculiar magnetoresistance picture showing the interference of MISO and PIRO develops with increasing temperature, when phonon-assisted transitions between Landau levels become important. The present study is motivated by this experiment, since the observed phenomenon requires a theoretical explanation. A more general aim of this paper is to present a theory of linear magnetoresistance in multisubband systems by taking into account interaction of electrons with both impurities and acoustic phonons. So far, the theoretical research of quantum magnetoresistance in such systems was limited by consideration of electron-impurity scattering.[22] The theory given below suggests that the interference phenomenon observed in two-subband quantum well[25] is caused by phonon-assisted intersubband scattering of electrons. Under Landau quantization, this scattering leads to magnetoresistance oscillations whose periodicity is governed by combined frequencies determined by characteristic phonon energy and subband splitting energy. Formally, the magnetoresistance contains intrasubband and intersubband quantum contributions caused by both scattering mechanisms. The intrasubband impurity-assisted scattering leads to monotonic positive quantum magnetoresistance, while the intersubband impurity-assisted scattering is responsible for

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MISO. The intrasubband phonon-assisted scattering leads to oscillating magnetoresistance similar to PIRO in single-subband systems.[10] Finally, the intersubband phonon-assisted scattering produces the MISO-PIRO interference contribution in magnetoresistance. The properties of this oscillating contribution are strongly influenced by the effect of suppression of intersubband transitions involving phonons with small transverse momenta.

The paper is organized as follows. Section II presents basic equations used for calculation of resistivity and the analytical results of this calculation. Section III contains comparison with experiment, discussion, and conclusions.

II. FORMALISM

The formalism for description of 2D electrons interacting with impurities and bulk acoustic phonons in the presence of a weak transverse magnetic field and external fields is based on the self-consistent Born approximation and given in detail in Refs. 10 and 26. These results can be straightforwardly extended to the case of multisubband occupation (see also Ref. 22 for electrons interacting with impurities). The kinetic equation for the generalized Wigner distribution function $f_{j\varepsilon\varphi}$ depending on the subband index j, energy ε, and electron momentum angle φ is written as

$$\omega_c \frac{\partial f_{j\varepsilon\varphi}}{\partial \varphi} = J^{im}_{j\varepsilon\varphi} + J^{ph}_{j\varepsilon\varphi}. \quad (1)$$

The electron-impurity (im) and electron-phonon (ph) collision integrals standing in this equation are

$$J^{im}_{j\varepsilon\varphi} = \sum_{j'} \int_0^{2\pi} \frac{d\varphi'}{2\pi} \nu_{jj'}(\varphi-\varphi') D_{j'\varepsilon+\gamma_{jj'}} [f_{j'\varepsilon+\gamma_{jj'}\varphi'} - f_{j\varepsilon\varphi}], \quad (2)$$

and

$$J^{ph}_{j\varepsilon\varphi} = \sum_{j'} \int_0^{2\pi} \frac{d\varphi'}{2\pi} \sum_{\lambda} \int_0^{\infty} \frac{dq_z}{\pi} m M^{jj'}_{\lambda\mathbf{Q}} \{[(N_{\omega_{\lambda\mathbf{Q}}} + f_{j\varepsilon\varphi}) f_{j'\varepsilon-\omega_{\lambda\mathbf{Q}}+\gamma_{jj'}\varphi'} - (N_{\omega_{\lambda\mathbf{Q}}}+1) f_{j\varepsilon\varphi}] D_{j'\varepsilon-\omega_{\lambda\mathbf{Q}}+\gamma_{jj'}}$$
$$+ [(N_{\omega_{\lambda\mathbf{Q}}} + 1 - f_{j\varepsilon\varphi}) f_{j'\varepsilon+\omega_{\lambda\mathbf{Q}}+\gamma_{jj'}\varphi'} - N_{\omega_{\lambda\mathbf{Q}}} f_{j\varepsilon\varphi}] D_{j'\varepsilon+\omega_{\lambda\mathbf{Q}}+\gamma_{jj'}}\}. \quad (3)$$

Here and below, the system of units with $\hbar=1$ is used.

The electron-impurity collisions are described in Eq. (2) by the scattering rates $\nu_{jj'}(\theta)=m w_{jj'}[q_{jj'}(\theta)]$, where $w_{jj'}(q)$ are the spatial Fourier transforms of the correlators of random scattering potential, m is the effective mass of electrons, $q_{jj'}(\theta)=\sqrt{p_j^2+p_{j'}^2-2p_j p_{j'} \cos\theta}$ is the momentum transferred in the elastic scattering, p_j is the Fermi momentum in the subband j, and $\theta=\varphi-\varphi'$ is the scattering angle. The subband-dependent Fermi momenta are $p_j=\sqrt{2m(\varepsilon_F-\varepsilon_j)}$, where ε_j are the subband energies relative to the reference point of the chemical potential ε_F. These momenta are related to partial electron densities in the subbands as $p_j^2=2\pi n_j$. The total sheet density in the 2D layer is $n_s=\Sigma_j n_j$.

The phonons are characterized by the mode index λ and three-dimensional phonon momentum $\mathbf{Q}=(\mathbf{q},q_z)$. The squared matrix element of electron-phonon interaction potential is represented as

$$M^{jj'}_{\lambda\mathbf{Q}} = C_{\lambda\mathbf{Q}} I_{jj'}(q_z), \quad I_{jj'}(q_z) = |\langle j|e^{iq_z z}|j'\rangle|^2. \quad (4)$$

The overlap factor $I_{jj'}$ depends on the form of envelope wave functions for the states $|j\rangle$ and $|j'\rangle$ of the corresponding subbands. The function $C_{\lambda\mathbf{Q}}$ is the squared matrix element in the bulk. This function includes both deformation potential and piezoelectric mechanisms of interaction, and its explicit form can be found in Ref. 10. The anisotropic phonon modes in cubic crystals are described by the eigenstate problem

$$\sum_j [K_{ii'}(\mathbf{Q}) - \delta_{ii'}\rho_M \omega^2] \mathrm{e}_{\lambda\mathbf{Q}i'} = 0, \quad (5)$$

where i and i' are the Cartesian coordinate indices, $K_{ii'}(\mathbf{Q})$ is the dynamical matrix expressed through three elastic constants, ρ_M is the material density, and $\mathrm{e}_{\lambda\mathbf{Q}i}$ are the components of the unit vector of the mode polarization. The solution of Eq. (5) gives the phonon frequencies $\omega=\omega_{\lambda\mathbf{Q}}$. The phonon system is assumed to be in equilibrium and is not affected by interaction with electrons, so $N_{\omega_{\lambda\mathbf{Q}}}=[\exp(\omega_{\lambda\mathbf{Q}}/T)-1]^{-1}$ in Eq. (3) is the Planck's distribution function. Owing to the smallness of phonon velocities compared to Fermi velocities, the electron-phonon scattering is treated in the quasielastic approximation with $Q=\sqrt{q_z^2+q_{jj'}^2(\theta)}$.

Finally, the Landau quantization of electron states is described in Eqs. (2) and (3) by the dimensionless (normalized to its value at $B=0$) subband-dependent density of states $D_{j\varepsilon}$, and the effect of external dc field is given by the function[22]

$$\gamma_{jj'}(\varphi,\varphi') = \frac{e}{2i\omega_c}[E_-(v_j e^{i\varphi} - v_{j'} e^{i\varphi'}) - E_+(v_j e^{-i\varphi} - v_{j'} e^{-i\varphi'})], \quad (6)$$

where $E_{\pm}=E_x \pm iE_y$, $\mathbf{E}=(E_x,E_y)$ is the dc field strength, and $v_j=p_j/m$ are the Fermi velocities.

The components of the current density are expressed through the distribution function as

$$\begin{pmatrix} j_x \\ j_y \end{pmatrix} = \frac{e}{\pi}\sum_j \int d\varepsilon D_{j\varepsilon} p_j \int_0^{2\pi} \frac{d\varphi}{2\pi}\begin{pmatrix} \cos\varphi \\ \sin\varphi \end{pmatrix} f_{j\varepsilon\varphi} + \sigma_\perp \begin{pmatrix} -E_y \\ E_x \end{pmatrix}, \tag{7}$$

where $\sigma_\perp = e^2 n_s / m\omega_c$ is the classical Hall conductivity. To find the linear response of the system to the applied electric field, one may expand $f_{j'\varepsilon+\gamma_{jj'}\varphi'}$ in the collision integrals as $f_{j'\varepsilon\varphi'} + (\partial f_\varepsilon / \partial \varepsilon)\gamma_{jj'}$, where f_ε is the equilibrium (Fermi-Dirac) distribution function. Then, the kinetic equation allows one to find the anisotropic distribution function $f_{j\varepsilon\varphi}$ proportional to the dc field strength. The basic Eqs. (1)–(7) are obtained for spatially homogeneous systems under assumption that the temperature, cyclotron energy, and collisional broadening energies (see Eq. (13) below) are small in comparison with ε_F. Also, the collisional broadening is assumed to be small compared to subband splitting energies $|\varepsilon_j - \varepsilon_{j'}|$.

In the regime of classically strong magnetic fields, when the cyclotron frequency is much larger than the transport scattering rate of electrons, the above equations lead to the following form of the diagonal (dissipative) conductivity:

$$\sigma_d = \frac{e^2 n_s}{m\omega_c^2}\sum_{jj'} \frac{n_j + n_{j'}}{2n_s}\left[\nu_{jj'}^{im} \int d\varepsilon \left(-\frac{\partial f_\varepsilon}{\partial \varepsilon}\right) D_{j\varepsilon} D_{j'\varepsilon} + \hat{\mathcal{S}}_{jj'} \left\{ \frac{T}{\omega_{\lambda \mathbf{Q}}^2} F\left(\frac{\omega_{\lambda \mathbf{Q}}}{2T}\right) \int d\varepsilon (f_{\varepsilon - \omega_{\lambda \mathbf{Q}}} - f_\varepsilon)(D_{j\varepsilon} D_{j'\varepsilon - \omega_{\lambda \mathbf{Q}}} + D_{j'\varepsilon} D_{j\varepsilon - \omega_{\lambda \mathbf{Q}}}) \right\}\right], \tag{8}$$

where $\nu_{jj'}^{im} = (2\pi)^{-1}\int_0^{2\pi} d\theta \nu_{jj'}(\theta)\mathcal{F}_{jj'}(\theta)$ are the impurity-assisted transport rates, $\mathcal{F}_{jj'}(\theta) = 1 - 2p_j p_{j'} \cos\theta / (p_j^2 + p_{j'}^2)$, and $F(x) = [x/\sinh(x)]^2$. The scattering of electrons by phonons is conveniently described by the integral operators $\hat{\mathcal{S}}_{jj'}$ defined as

$$\hat{\mathcal{S}}_{jj'}\{A\} \equiv \int_0^{2\pi} \frac{d\theta}{2\pi}\int_0^{2\pi} \frac{d\phi}{2\pi}\sum_\lambda \int_0^\infty \frac{dq_z}{\pi} m M_{\lambda \mathbf{Q}}^{jj'} \mathcal{F}_{jj'}(\theta) A, \tag{9}$$

where $\phi = (\varphi + \varphi')/2$ and A is a function of the variables of integration. The quantities $\omega_{\lambda \mathbf{Q}}$ and $M_{\lambda \mathbf{Q}}^{jj'}$ depend on the transverse phonon momentum q_z, scattering angle θ, and polar angle of the vector $\mathbf{q}$ (this angle is equal to $\pi/2 + \phi$) as described by Eqs. (4) and (5), and relation $Q = \sqrt{q_z^2 + q_{jj'}^2(\theta)}$. The diagonal resistivity ρ_d is expressed through σ_d as $\rho_d \simeq \sigma_d / \sigma_\perp^2$. Under the condition $2\pi^2 T \gg \omega_c$, when the SdHO are thermally averaged out, the resistivity is given by

$$\rho_d = \frac{m}{e^2 n_s}\sum_{jj'} \frac{n_j + n_{j'}}{2n_s}\left[\nu_{jj'}^{im} \langle D_{j\varepsilon} D_{j'\varepsilon}\rangle_\varepsilon + \hat{\mathcal{S}}_{jj'} \left\{ \frac{T}{\omega_{\lambda \mathbf{Q}}} F\left(\frac{\omega_{\lambda \mathbf{Q}}}{2T}\right) \times (\langle D_{j\varepsilon} D_{j'\varepsilon - \omega_{\lambda \mathbf{Q}}}\rangle_\varepsilon + \langle D_{j'\varepsilon} D_{j\varepsilon - \omega_{\lambda \mathbf{Q}}}\rangle_\varepsilon) \right\}\right], \tag{10}$$

where the symbol $\langle \ldots \rangle_\varepsilon$ denotes the averaging over the interval of cyclotron energy (the density of states $D_{j\varepsilon}$ is periodic in energy with the period ω_c).

The expression (10) applies to systems with arbitrary number of subbands and is valid for arbitrary (periodic) density of states. If quantum oscillations of the density of states are neglected ($D_{j\varepsilon} = 1$), this expression produces the classical resistivity $\rho_d^c = m\nu_{tr}/e^2 n_s$, where ν_{tr} is the transport scattering rate comprising contributions from both scattering mechanisms

$$\nu_{tr} = \sum_{jj'} \frac{n_j + n_{j'}}{2n_s}\left[\nu_{jj'}^{im} + \hat{\mathcal{S}}_{jj'} \left\{ \frac{2T}{\omega_{\lambda \mathbf{Q}}} F\left(\frac{\omega_{\lambda \mathbf{Q}}}{2T}\right) \right\}\right]. \tag{11}$$

The consideration below is carried out for two-subband systems. The equations for calculation of the density of states in these systems can be found in Ref. 20. An analytical averaging in Eq. (10) is done under approximation of overlapping Landau levels, when only the first oscillatory harmonics of the density of states are retained. The result represents a sum of the classical resistivity and quantum contributions caused by both intrasubband and intersubband scattering

$$\rho_d = \rho_d^c + \frac{m}{e^2 n_s}\left[d_1^2 \frac{2n_1}{n_s}\mathcal{V}_{11} + d_2^2 \frac{2n_2}{n_s}\mathcal{V}_{22} + 2d_1 d_2 \mathcal{V}_{12} \cos\frac{2\pi\Delta}{\omega_c}\right], \tag{12}$$

where $\Delta = \varepsilon_2 - \varepsilon_1$ is the subband splitting energy, $d_j = \exp(-\pi\nu_j/\omega_c)$ are the Dingle factors and

$$\nu_j = \int_0^{2\pi} \frac{d\theta}{2\pi}[\nu_{jj}(\theta) + \nu_{12}(\theta)] + \nu_j^{ee} \tag{13}$$

are the quantum relaxation rates determining the collisional broadening of Landau levels. These rates comprise contributions due to electron-impurity and electron-electron (ee) interaction (see the next section for evaluation of ν_j^{ee}). The phonon-assisted scattering contribution to ν_j is neglected because it is small compared to the contributions included in Eq. (13). The oscillating scattering rates $\mathcal{V}_{jj'}$ are introduced as

$$\mathcal{V}_{jj'} = \nu_{jj'}^{im} + \hat{\mathcal{S}}_{jj'} \left\{ \frac{2T}{\omega_{\lambda \mathbf{Q}}} F\left(\frac{\omega_{\lambda \mathbf{Q}}}{2T}\right) \cos\frac{2\pi\omega_{\lambda \mathbf{Q}}}{\omega_c}\right\}. \tag{14}$$

The phonon-induced parts of the rates $\mathcal{V}_{jj'}$ describe PIRO in multisubband systems. Since the intersubband PIRO component entering the last term in Eq. (12) is multiplied by the MISO factor $\cos(2\pi\Delta/\omega_c)$, the corresponding contribution describes the interference of MISO and PIRO. The conven-

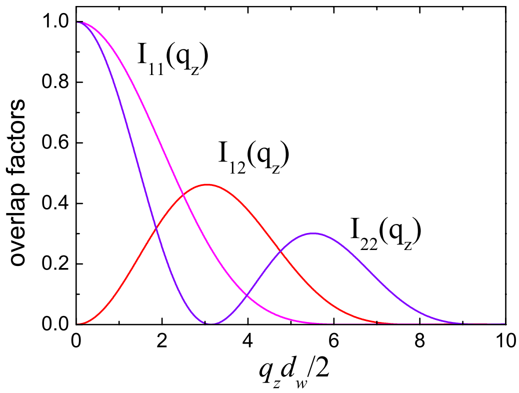

FIG. 1. (Color online) Overlap factors $I_{jj'}(q_z)$ for rectangular well model, according to Eq. (16).

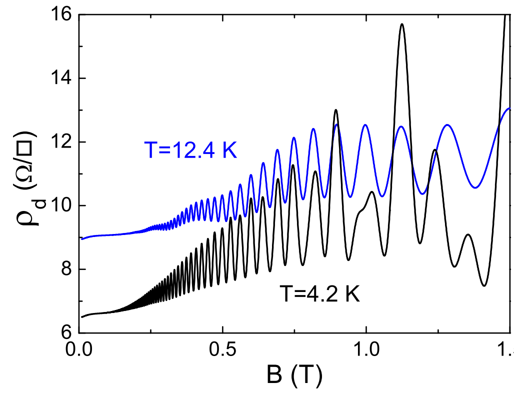

FIG. 2. (Color online) Calculated magnetoresistance of two-subband quantum well studied in Ref. 25 at $T=4.2$ K and 12.4 K.

tional MISO come from the impurity-assisted intersubband contribution proportional to ν_{12}^{im}. The intrasubband quantum contributions in Eq. (12) do not oscillate with Δ.

The SdHO correction to the resistivity can be obtained with the use of Eq. (8)

$$\delta\rho_d = -\mathcal{T}\frac{2m}{e^2 n_s}\sum_{j=1,2}\left[\frac{2n_j}{n_s}\widetilde{\mathcal{V}}_{jj}+\widetilde{\mathcal{V}}_{12}\right]d_j\cos\frac{2\pi\varepsilon_{Fj}}{\omega_c}, \quad (15)$$

where $\mathcal{T}=(2\pi^2 T/\omega_c)/\sinh(2\pi^2 T/\omega_c)$ is the thermal suppression factor and $\varepsilon_{Fj}=\varepsilon_F-\varepsilon_j=\pi n_j/m$ are the Fermi energies in the subbands. The rates $\widetilde{\mathcal{V}}_{jj'}$ differ from $\mathcal{V}_{jj'}$ of Eq. (14) by the substitution of $\sin(2\pi\omega_{\lambda\mathbf{Q}}/\omega_c)/(2\pi\omega_{\lambda\mathbf{Q}}/\omega_c)$ in place of $\cos(2\pi\omega_{\lambda\mathbf{Q}}/\omega_c)$ in the argument of $\hat{\mathcal{S}}_{jj'}$. The expression (15) includes contributions due to both impurity-assisted and phonon-assisted scattering. Usually, the latter contribution is less significant, because the activation of phonon-assisted scattering takes place at the temperatures when SdHO are suppressed in the weak magnetic field region. Nevertheless, the phonon-assisted SdHO may become important in the samples of very high purity, where relative contribution of phonon-assisted scattering to magnetotransport remains essential in the interval of parameters corresponding to $\mathcal{T}\sim 1$.

III. RESULTS AND DISCUSSION

The results of the previous section are applied below for calculation of the magnetoresistance of two-subband electron systems based on GaAs layers grown along the [001] crystallographic axis. The consideration is focused on two-subband single quantum well of width d_w. Approximating the shape of the confinement potential by a deep rectangular well, one can write the overlap factors for electron-phonon interaction as $I_{jj'}(q_z)=\mathcal{I}_{jj'}^2(q_z d_w/2)$, where

$$\mathcal{I}_{11}(x)=\frac{\sin x/x}{1-(x/\pi)^2}, \quad \mathcal{I}_{22}(x)=\frac{\sin x/x}{1-(x/2\pi)^2},$$

$$\mathcal{I}_{12}(x)=\frac{32}{9\pi^2}\frac{x\cos x}{[1-(2x/\pi)^2][1-(2x/3\pi)^2]}. \quad (16)$$

Figure 1 presents $I_{jj'}(q_z)$ in the graphic form.

To describe the elastic scattering, one needs to specify the impurity-potential correlator $w_{jj'}(q)$, which depends on numerous parameters such as nature of the impurities, distribution of the impurities over the structure, properties of interface roughness (if present), etc. Thus, the exact form of $w_{jj'}(q)$ is generally unknown. For calculations, $w_{jj'}(q)$ is modeled as $w_{11}(q)=w_{22}(q)=w_0 e^{-l_c q}$ and $w_{12}(q)=w_0\chi_q e^{-l_c q}$ with $\chi_q=(qd_w/\pi)^2/[1+(qd_w/\pi)^2]$, where l_c is the correlation length determining the spatial scale of the random scattering potential. In high-mobility (modulation-doped) structures, this potential is smooth: l_c is larger than the Fermi wavelength. The factor χ_q is introduced to take into account suppression of intersubband scattering by a smooth random potential at small transferred momentum, $q<\pi/d_w$, as follows from orthogonality of the wave functions belonging to different subbands.

To estimate the electron-electron scattering rates ν_j^{ee} contributing into the Dingle factors, one may use the results of Refs. 27 and 28. Since in single quantum wells the subband separation is large, the intersubband electron-electron scattering requires a large momentum transfer and is considerably suppressed at low temperatures in comparison with intrasubband (small-angle) scattering. Therefore, it is reasonable to use the corresponding single-subband expressions, $\nu_j^{ee}\simeq\lambda_j T^2/\varepsilon_{Fj}$. The coefficients λ_j are of the order of unity and estimated as[27,28] $\lambda_j\simeq\pi^{-1}\ln(q_0 v_j/T)$, where $q_0=2/a_B$ is the inverse screening length (a_B is the Bohr radius).

Figure 2 shows the diagonal resistivity calculated for a quantum well investigated in Ref. 25 ($d_w=26$ nm, $n_1=6.24\times10^{11}$ cm^{-2}, $n_2=1.91\times10^{11}$ cm^{-2}, $\Delta=15.5$ meV) at two temperatures used in the experiment. The calculation is based on the expression (12) including the SdHO correction (15), the latter is important at $T=4.2$ K in the region above 0.5 T. An additional term for classical magnetoresistance (essential in the region of $B<0.05$ T) has been included according to Ref. 29. The parameters describing the acoustic phonon spectrum and electron-phonon interaction can be found in Ref. 10. The plots show a good agreement with experiment as concerns both the background and oscillating resistivity. The small-period oscillations are the MISO. The oscillating modifications of the resistivity by electron-phonon interaction (large-period features) are seen already at $T=4.2$ K. These features become pronounced with increas-

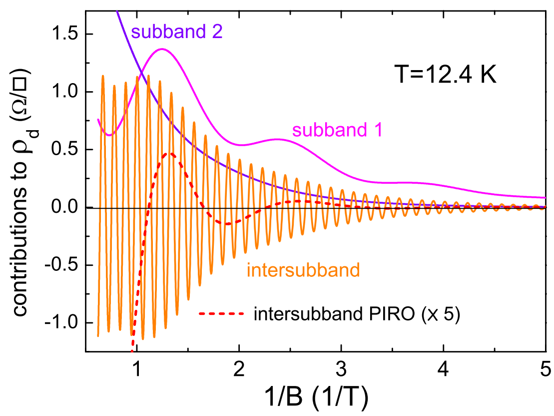

FIG. 3. (Color online) Quantum contributions to resistivity at $T=12.4$ K. Solid lines show the three terms of Eq. (12) as functions of the inverse magnetic field. Dashed line is the phonon-induced part of the intersubband term without the MISO factor $\cos(2\pi\Delta/\omega_c)$ (this part is multiplied by 5 for clarity).

ing temperature, when more phonons participate in the transitions. The "irregular" features at $T=4.2$ K in the high-field region, seen also in the experimental graphs, are caused by superposition of MISO and SdHO. Notice that the calculations involve no adjustable parameters for electron-phonon interaction. The only adjustable parameter of the theory, the correlation length of the impurity potential (l_c), is determined from fitting the calculated amplitude of quantum oscillations of resistivity to experimental data.

Figure 3 shows behavior of three quantum contributions to resistivity entering Eq. (12). The contribution caused by electron transitions within the first subband shows distinct PIRO with periodicity determined by a characteristic phonon energy $2p_1s$, where $s\simeq 5.2$ km/s corresponds to the velocity of the longitudinal acoustic phonon mode in GaAs. This correlation is understandable, since PIRO in high-density samples occur mainly due to scattering by longitudinal acoustic phonons.[5,10] The second-subband contribution is suppressed at small B because both electron-impurity and electron-electron scattering for this subband are stronger, which leads to a considerable difference in Dingle factors: $d_2 \ll d_1$. Since p_2 is approximately twice smaller than p_1, the PIRO associated with the second subband fall into the region of small B, remain feeble, and are not seen in the plot. Finally, the intersubband contribution represents MISO whose amplitude is nontrivially varied by the oscillating part of $\mathcal{V}_{12}$ caused by intersubband phonon-assisted transitions. Nonmonotonic changes of MISO amplitude occur around $B \simeq 1$ T, while less distinct changes are visible at lower magnetic field, in agreement with experimental observations. One may say that MISO are enveloped by intersubband PIRO. This intersubband PIRO term is shown in Fig. 3 by the dashed line.

The period of intersubband PIRO is very close to that of the first-subband PIRO. At a first glance, this coincidence seems to be surprising, because the Fermi momenta in the subbands differ considerably, so the correlation between PIRO period and Fermi momentum established[3] for single-subband systems does not work. In particular, since the main contribution to oscillating phonon-induced resistivity comes from scattering angles around $\theta=\pi$ (backscattering) and transverse phonon momenta around $q_z=0$ (see Ref. 10 for the detailed analysis), one may expect that intersubband PIRO periodicity is governed by the phonon energy $(p_1+p_2)s$, which is smaller than $2p_1s$. However, for intersubband scattering the situation is more complicated, because intersubband transitions with small q_z are strongly suppressed by the overlap factor $I_{12}(q_z)$. The latter is proportional to q_z^2 at $q_z \ll \pi/d_w$, due to the wave function orthogonality. Therefore, a more correct estimate for characteristic phonon energy is $\sqrt{(p_1+p_2)^2+q_{z0}^2}s$, where q_{z0} is a (nonzero) transverse momentum giving the main contribution to the integral over q_z in $\hat{\mathcal{S}}_{12}\{\ldots\}$ of Eq. (14). Notice that q_{z0} decreases with increasing well width d_w and is sensitive to magnetic field (q_{z0} decreases with increasing order of magnetophonon resonance). Thus, the intersubband PIRO are not expected to be periodic as a function of $1/B$, even if a single phonon mode is considered. Another consequence of the constraint introduced by the overlap factor is a suppression of intersubband PIRO amplitudes. The suppression becomes stronger for higher-order magnetophonon resonances in the region of weak magnetic fields. A rough estimate for q_{z0} in the region of low-order magnetophonon resonances can be based on the fact that $I_{12}(q_z)$ has a maximum at $q_zd_w\simeq 6$ (Fig. 1). Substituting $q_{z0}=6/d_w$ into the phonon energy $\sqrt{(p_1+p_2)^2+q_{z0}^2}s$, one gets the result close to $2p_1s$, which explains the coincidence of the positions of the first peaks of intersubband and intrasubband PIRO under conditions of the experiment Ref. 25. This coincidence is accidental: for a quantum well with a different width d_w or a different electron density n_s, the positions of these peaks should be different. The above consideration also shows that the amplitude of intersubband PIRO is suppressed with decreasing d_w.

The interference of MISO and PIRO is clear both in experimental and in theoretical magnetoresistance in spite of the weakness of intersubband phonon-assisted scattering responsible for this effect. In principle, the electron-phonon interaction is capable to produce inversion (flip) of MISO peaks (such an effect is typical for the experiments involving microwave irradiation).[21,22] Formally, the flip occurs when $\mathcal{V}_{12}$ becomes negative, see Eqs. (12) and (14). To reach this condition, one should study samples of higher purity, where oscillating rate due to intersubband phonon-assisted scattering (second term in $\mathcal{V}_{12}$) may exceed by amplitude the impurity-assisted intersubband scattering rate ν_{12}^{im}. Theoretical estimates show that for the quantum well system studied in Ref. 25 the MISO flip is attainable at the fields above 1 T if the low-temperature mobility is increased to 4×10^6 cm^2/V s. Notice that the expression (12) is applicable even in the case when elastic impurity-assisted scattering is absent, so the resistivity is entirely determined by electron-phonon interaction (the Dingle factors in this case are determined by electron-electron interaction only). The corresponding magnetoresistance is plotted in Fig. 4 together with quantum contributions due to intrasubband and intersubband scattering. One can see that the oscillating resistivity at weak magnetic fields in dominated by first-subband PIRO, while the intersubband PIRO becomes essential starting from 0.3 T. The interference of MISO and PIRO leads to inversion of the

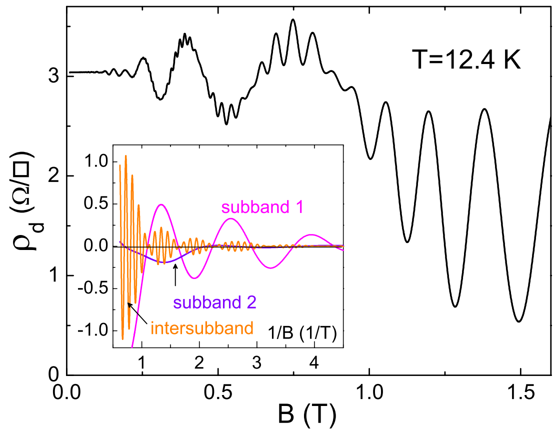

FIG. 4. (Color online) Magnetoresistance of two-subband quantum well from Ref. 25, calculated for the case when elastic scattering is absent. The inset shows quantum contributions to resistivity, similar as in Fig. 3.

groups of MISO peaks around 0.5 T and above 1 T. Although the situation with no elastic scattering does not occur in realistic samples, it is interesting from the theoretical point of view, because the interference oscillations exist here in the pure form (see a number of distinct nodes in the intersubband contribution plotted in the inset to Fig. 4).

The description of magnetoresistance given by Eq. (12) can be also applied for double quantum wells, where two-subband electron system is formed as a result of tunnel hybridization of the ground states in the adjacent wells. The overlap factors for electron-phonon scattering in this case are different from those of Eq. (16). Assuming that both layers are described as rectangular wells of equal width d_w and the system is balanced (electron densities in the wells are equal), one obtains

$$I_{11}(q_z) = I_{22}(q_z) = \mathcal{I}_{11}^2(q_z d_w/2)\cos^2(q_z Z/2),$$

$$I_{12}(q_z) = \mathcal{I}_{11}^2(q_z d_w/2)\sin^2(q_z Z/2), \tag{17}$$

where Z is the distance between the centers of the wells. The geometric interference factors oscillating with $q_z Z$ appear because of tunnel coherence of electron states and may lead to magneto-oscillations of interwell current[30] when emission of acoustic phonons with large q_z is favorable (such oscillations are not expected to be essential in the magnetoresistance studied in this paper). Similar to the case of single well, the intersubband overlap factor is suppressed at $q_z \ll \pi/Z$. From qualitative point of view, the magnetoresistance of double quantum wells is similar to that of the two-subband single quantum wells. However, since the subband separation energy Δ in double quantum wells is considerably smaller, the periods of MISO and PIRO can be made comparable.

In conclusion, a microscopic theory of magnetoresistance oscillations in multisubband systems is presented. The role of electron scattering by acoustic phonons is studied in detail. Owing to phonon-assisted transitions between Landau levels belonging to different subbands, the magnetoresistance shows the interference of magnetointersubband oscillations with magnetophonon oscillations. The results are in agreement with recent experimental data[25] on high-mobility quantum wells with two occupied subbands.

[1] C. L. Yang, J. Zhang, R. R. Du, J. A. Simmons, and J. L. Reno, Phys. Rev. Lett. **89**, 076801 (2002).
[2] M. A. Zudov, R. R. Du, J. A. Simmons, and J. L. Reno, Phys. Rev. B **64**, 201311(R) (2001).
[3] M. A. Zudov, I. V. Ponomarev, A. L. Efros, R. R. Du, J. A. Simmons, and J. L. Reno, Phys. Rev. Lett. **86**, 3614 (2001).
[4] J. Zhang, S. K. Lyo, R. R. Du, J. A. Simmons, and J. L. Reno, Phys. Rev. Lett. **92**, 156802 (2004).
[5] A. A. Bykov, A. K. Kalagin, and A. K. Bakarov, JETP Lett. **81**, 523 (2005).
[6] W. Zhang, M. A. Zudov, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. **100**, 036805 (2008).
[7] X. L. Lei, Phys. Rev. B **77**, 205309 (2008).
[8] A. T. Hatke, M. A. Zudov, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. **102**, 086808 (2009).
[9] A. A. Bykov and A. V. Goran, JETP Lett. **90**, 578 (2009).
[10] O. E. Raichev, Phys. Rev. B **80**, 075318 (2009).
[11] V. Polyanovsky, Fiz. Tekh. Poluprovodn. **22**, 2230 (1988) [Sov. Phys. Semicond. **22**, 1408 (1988)].
[12] P. T. Coleridge, Semicond. Sci. Technol. **5**, 961 (1990).
[13] D. R. Leadley, R. Fletcher, R. J. Nicholas, F. Tao, C. T. Foxon, and J. J. Harris, Phys. Rev. B **46**, 12439 (1992).
[14] T. H. Sander, S. N. Holmes, J. J. Harris, D. K. Maude, and J. C. Portal, Phys. Rev. B **58**, 13856 (1998).
[15] A. C. H. Rowe, J. Nehls, R. A. Stradling, and R. S. Ferguson, Phys. Rev. B **63**, 201307(R) (2001).
[16] N. C. Mamani, G. M. Gusev, T. E. Lamas, A. K. Bakarov, and O. E. Raichev, Phys. Rev. B **77**, 205327 (2008).
[17] A. A. Bykov, D. P. Islamov, A. V. Goran, and A. I. Toropov, JETP Lett. **87**, 477 (2008).
[18] M. E. Raikh and T. V. Shahbazyan, Phys. Rev. B **49**, 5531 (1994).
[19] N. S. Averkiev, L. E. Golub, S. A. Tarasenko, and M. Willander, J. Phys.: Condens. Matter **13**, 2517 (2001).
[20] O. E. Raichev, Phys. Rev. B **78**, 125304 (2008).
[21] S. Wiedmann, G. M. Gusev, O. E. Raichev, T. E. Lamas, A. K. Bakarov, and J. C. Portal, Phys. Rev. B **78**, 121301(R) (2008).
[22] S. Wiedmann, N. C. Mamani, G. M. Gusev, O. E. Raichev, A. K. Bakarov, and J. C. Portal, Phys. Rev. B **80**, 245306 (2009).
[23] W. Zhang, M. A. Zudov, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. **98**, 106804 (2007).
[24] A. T. Hatke, H.-S. Chiang, M. A. Zudov, L. N. Pfeiffer, and K. W. West, Phys. Rev. B **77**, 201304(R) (2008).
[25] A. Bykov, A. Goran, and S. Vitkalov, Phys. Rev. B **81**, 155322 (2010).
[26] O. Raichev, Phys. Rev. B **81**, 165319 (2010).
[27] G. F. Giuliani and J. J. Quinn, Phys. Rev. B **26**, 4421 (1982).
[28] Y. Berk, A. Kamenev, A. Palevski, L. N. Pfeiffer, and K. W. West, Phys. Rev. B **51**, 2604 (1995).
[29] N. C. Mamani, G. M. Gusev, E. C. F. da Silva, O. E. Raichev, A. A. Quivy, and A. K. Bakarov, Phys. Rev. B **80**, 085304 (2009).
[30] O. E. Raichev and F. T. Vasko, Phys. Rev. B **50**, 12199 (1994).